Due to the U.S. Interior Secretary's recent approval for natural gas exploration, our company has recently started drilling in Vernal, Utah. We are now faced with running some pipeline from the drill site to our closest refinery. This is easier said than done, there are a few potential costly obstacles in our way. There is a mountain directly east of the drill site and the shortest pathway from the drill site to the refinery is across private ground, which incurs an extra right-of-way fee. The last two logical options are to run the pipeline around the privately owned ground, on BLM ground, or one that hasn't been discovered yet. I have been given the task of finding the most cost efficient route from these possibilities.

First off lets explore the facts. The cost for materials and labor fees to run the pipeline across BLM ground is $\$ 500,000$ per mile. For any pipeline run across private ground, our company incurs an additional $\$ 350,000$ per mile right-of-way fee, equaling $\$ 850,000$ per mile. The mountain directly east of the well will have to be drilled through in order to run the pipeline due east. This will set the company back $\$ 1,200,000$ on top of the normal costs of the pipeline. It will also require an environmental impact study that is estimated to cost $\$ 240,000$ and will delay the project by four months, costing another $\$ 140,000$ per month.

Now that all the information has been absorbed lets discuss each scenario and the math involved in each. First off lets discuss the cost of running pipeline strictly on BLM ground. There are two options for running pipeline on BLM ground. The first is around the private ground with only pipeline as cost. The other is through the mountain around and then down the side of the border of privately owned ground.

The private ground encompasses an area of eight by twenty-five, or 200 square miles of land. Our drill site is 3 miles east off the border of this land. To go around it we will have to travel three miles due west, then eight miles due south, and then twenty-five miles due east to finally arrive at our refinery. To solve this it is just basic algebra and not to exciting. The total miles traveled, thirty-six, times the cost, $\$ 500,000$ per mile, giving us a grand total of $\$ 18,000,000$.

Cost $=(3+8+25) 500,000=18,000,000$


At first glance, the path through the mountain seems like it would not be cost effective, but once all the numbers are crunched it might surprise you. Lets start with the basics and remove the mountain from the equation. Without the mountain we would have
to run the pipeline twenty-two miles due east and then eight miles due south across the perimeter of the privately owned land to the refinery from the drill site. This gives us a grand total of thirty miles of pipeline to run at $\$ 500,000$ per mile. Then we add the cost of $\$ 1,200,000$ for drilling through the mountain, the estimated cost of $\$ 240,000$ for the impact study, and the loss incurred of $\$ 560,000$ for the four months that the project will be delayed due to the impact study. All of this comes together to form a grand total of \$17,000,000.

Cost $=(22+8) 500,000+1,200,000+240,000+(4) 140,000=17,000,000$


The next scenario that we will be looking at is the cost of running the pipeline the shortest distance through the privately owned ground to the refinery. First off we have to discuss the principle that is being used to come to my conclusion, the Pythagorean theorem. The theorem is a statement about triangles containing a right angle. To be specific the theorem states that: "The area of the square built upon the hypotenuse, the longest side, of a right triangle is equal to the sum of the areas of the squares up on the remaining sides." To be put a bit more clearly, the longest side of a right triangle can be found by taking the square root of both of its shorter sides squared and added together.

This is exactly what I did to find the shortest distance across the private ground. We know that the land is eight miles by twenty-five miles. We also know that the drill site is three miles south from the border of the land. This gives us one leg of our triangle that is eight miles and another that is twenty-two miles, twenty-five miles of border minus the three miles difference for the drill site. Because of the theorem stated above we can find the shortest distance by simply squaring both of those numbers, add them together, and then taking the square root. Thankfully it is not as difficult as it might sound. Eight squared is sixty-four and twenty-two squared is 484, both of these added together gives us 548 . Since this number is not completely factorable when square rooted it is best to leave it as it is to get the most accurate cost. All that is left is to times our square root by the cost of running our pipeline across private ground, 850,000 . This gives us a grand total of $\$ 19,897,989.85$.

Cost $=(500,000+350,000)\left(8^{\wedge} 2+22^{\wedge} 2\right)^{\wedge} 1 / 2=19,897,989.85$


The last scenario that I was asked to determine is the optimal place to run the pipeline to minimize cost. Usually the most direct path would be the least amount of cost,
but as proven above it is actually the most expensive. The cheapest is actually through the mountain, surprisingly, but will cost an extra four months of lost time. This leaves us hoping that there is another way to run the pipeline that will be faster and cheaper, luckily there is. I started by assuming that the most cost efficient way to get to the refinery would be to run the least amount of pipeline. After finding that the straight line from point A , being the drill sit, and point B , being the refinery, was not going to work; I figured there was a happy medium in between the eight mile expanse of the property and the shortest distance. So we know that the cost for going over private land is $\$ 850,000$ and going over BLM ground is $\$ 500.000$. We can assume that at some point to minimize cost we will have to run pipeline both across private land and BLM ground.

To find the unknown distance across private land we can set up the equation similar to how we found the shortest distance from point $A$ to point $B$. The hypotenuse, which we will refer to as H , is going to be based off of the eight miles expanse of the private property that does not change. We know that H is going to be found by squaring the two smaller sides, one of these being eight and the other that is unknown as of now. Lets refer to this unknown as X . With all of this information we now know that distance we need to travel across private land, H , is going to equal eight squared plus X squared.

The distance across BLM ground is not as hard to come across. As we look at X from the previous paragraph we can logically assume a few things. That it is going to somehow relate to the hypotenuse we are looking for and that it is going to be located somewhere within the twenty-two mile expanse, the twenty-five mile border minus the three miles difference the drill site is located from the edge of the property. This helps us get the last bit of information that we need to start our equation. Since X is located in our
first H equation we know it has to be subtracted from the equation we are working on currently. This can be stated as simply as twenty-two, miles of distanced need to be traveled, minus X.

With all of this information we can set up our starting point to figure out our minimum cost. This equation is going to be cost is equal to H times $\$ 850,000$ plus twenty-two minus $X$ times by $\$ 500,000$. To continue we need to discuss Derivatives. The definition of a derivative can be approached in two ways. One is geometrical, as a slope of a curve, and the other is physical, as a rate of change. Either way you approach it this idea gives an easier way to evaluate an equation with out plugging it into a graph. It basically will allow us to view the highest and lowest points possible for the equation to still be able to exist. What we are looking for is the lowest point, or our lowest cost.

The easiest way for me to explain the calculus involved is to show the step-bystep procedures involved in the calculations, which can be viewed at the bottom of the paragraph and then explain the steps. The first part is just the visual equation of what was talked about. What follows is a bit confusing, $\wedge 2$ are just stating that I am squaring both of the numbers. The ${ }^{\wedge} 1 / 2$ is the power notation for a square root, it is written like this to make it easier to calculate the derivative. To show that I am taking the derivative it is noted by Cost'. After taking the derivative I then solve for X by setting the equation equal to 0 . The solutions for X will give me all the points that are going to be maximum and minimum values. We are only looking for minimum cost so I will focus on the lowest number 5.814.

Cost $=850,000 \mathrm{H}+500,000(22-\mathrm{X})$
Cost $=850,000\left(8^{\wedge} 2+X^{\wedge} 2\right)^{\wedge} 1 / 2+500,000(22-X)$

$$
\begin{aligned}
\text { Cost }= & 850,000\left(64+X^{\wedge} 2\right)^{\wedge} 1 / 2+500,000(22-X) \\
\text { Cost' }^{\prime}= & 1 / 2\left(64+X^{\wedge} 2\right)^{\wedge} 1 / 2(2 X)(850,000)+500,000(-1) \\
\text { Cost' }^{\prime}= & 850,000 \mathrm{X} /\left(64+X^{\wedge} 2\right)^{\wedge} 1 / 2-500,000=0 \\
& 850,000 \mathrm{X} /\left(64+X^{\wedge} 2\right)^{\wedge} 1 / 2=500,000 \\
& 850,000 \mathrm{X} / 500,000=\left(64+X^{\wedge} 2\right)^{\wedge} 1 / 2 \\
& \left(850,000 \mathrm{X} / 500,000=\left(64+X^{\wedge} 2\right)^{\wedge} 1 / 2\right)^{\wedge} 2 \\
& 289 X^{\wedge} 2 / 100(\text { simplified })=64+X^{\wedge} 2 \\
& 289 X^{\wedge} 2 / 100-X^{\wedge} 2=64 \\
& 189 X^{\wedge} 2 / 100=64 \\
& 189 X^{\wedge} 2=6400 \\
& X^{\wedge} 2=33.862 \\
& X=5.819
\end{aligned}
$$

Now that we know what X is we can plug it into our original equations. The distance across private ground equals the square root of sixty-four plus 33.862 . The distance across BLM ground will be twenty-two minus 5.819. This leaves us knowing that if we run pipeline across private ground at a distance of 9.849 miles and run pipeline 16.181 miles across BLM ground we will have a minimum cost of $\$ 16,498,700$.


